A Theory of Frozen Light According to General Relativity

Dmitri Rabounski and Larissa Borissova

Abstract: We suggest a theory of frozen light, which was first registered in 2000 by Lene Hau, who pioneered this experimental research, which was then approved by two other groups of experimentalists. Frozen light is explained here as a new state of matter, which differs from the others (solid, gas, liquid, plasma). The explanation is given through space-time terms of the General Theory of Relativity, employing the mathematical apparatus of chronometric invariants (physically observable quantities) which are the respective projections of space-time quantities onto the line of time and the three-dimensional spatial section of an observer. We suggest to consider a region of space (space-time), where the metric is fully degenerate. It is shown that this is the ultimate case of the isotropic region (home of light-like massless particles, e.g. photons), where the metric is particularly degenerate so that the space-time interval is zero, while the observable time and three-dimensional intervals are nonzero and equal to each other. Both the space-time interval, the observable time interval, and the observable three-dimensional interval are zero in a fully degenerate region. This means that, from the point of view of a regular observer, any particle of a fully degenerate region travels instantly. Therefore, we refer to such a region and the particles inhabiting it as zero-space and zero-particles. Moving to coordinate quantities inside zero-space shows that the real speed therein is that of light, depending on the gravitational potential and the rotation of space. It is shown that the eikonal equation for zero-particles, expressed through physically observable quantities, is a standing wave equation: zero-particles appear to a regular "external" observer as standing light waves (stopped, or frozen light), while zero-space is filled with a system of standing light waves (light-like holograms). In the internal reference frame of zero-space, momentum does not conserve. This is solely a property of virtual photons of Quantum Electrodynamics. Therefore zero-particles (we can observe them as standing light waves) should play a rôle of virtual photons. Thus the frozen light experiments are an experimental "foreword" to discovery of zero-particles, which are virtual photons.

A thesis of this presentation has been submitted to the *APS March Meeting 2011*, planned on March 21–25, 2011, in Dallas, Texas.

Contents:

§1. Frozen light. An introduction. In the summer of 2000, Lene V. Hau, who pioneered light-slowing experiments over many years in the 1990's at Harvard University, first obtained light slowed down to rest state. In her experiment, light was stored, for milliseconds, in ultracold atoms of sodium (with a gaseous cloud of the atoms cooled down to within a millionth of a degree of absolute zero). This state was then referred to as *frozen light* or *stopped light*. An anthology of the primary experiments is given in her publications [1–5]. After the first success of 2000, Lene Hau still continues the study: in 2009, light was stopped for 1.5 second at her laboratory [6].

Then frozen light was approved, during one year, by two other groups of experimentalists. A group headed by Ronald L. Walsworth and Mikhail D. Lukin of the Harvard-Smithsonian Center for Astrophysics stopped light in a room-temperature gas [7]. In experiments conducted by Philip R. Hemmer at the Air Force Research Laboratory in Hanscom (Massachusetts), light was stopped in a cooled-down solid [8].

The best-of-all survey of all experiments on this subject was given in Lene Hau's *Frozen Light*, which was first published in 2001, in *Scientific American* [4]. Then an extended version of this paper was reprinted in 2003, in a special issue of the journal [5].

On the other hand, the frozen light problem was met by our theoretical study of the 1990's, which was produced independently of the experimentalists (we knew nothing about the experiments until January 2001, when the first success in stopping light was widely advertised in the scientific press). Our task was to reveal what kinds of particles could theoretically inhabit the space (space-time) of the General Theory of Relativity. We have obtained that, aside for mass-bearing and massless (light-like) particles, those of the third kind may also exist. Such particles inhabit a space with a fully degenerate metric, which is the ultimate case of the light-like (particularly degenerate) space. This means that the particles are the ultimate case of photons. It was shown that, from the viewpoint of a regular observer, they should be perceived as standing light waves (or frozen light, in other words).

These theoretical results were presented, among the others, in our book [9], which was first published in 2001 and then reprinted in 2008. However they were very fragmented along the book, where many problems (such as geodesic motion, gravitational collapse, and others) were discussed commonly for all particles. Therefore we have decided to join the results in this single paper, thus giving a complete presentation of our theory of frozen light.

§2. Introducing fully degenerate space (zero-space) as the ultimate case of (particularly degenerate) light-like space. Once we want to reveal a descriptive picture of any physical theory, we need to express the results through real physical quantities (physical observables), which can be measured in experiments. In the General Theory of Relativity, a complete mathematical apparatus for calculating physically observable quantities was introduced in 1944 by Abraham Zelmanov [10, 11], and is known as the *theory of chronometric invariants*. Its essence consists of projecting four-dimensional quantities onto the line of time and the three-dimensional spatial section of an observer. As a result, we obtain quantities observable in practice.

Expressing the four-dimensional (space-time) interval through physically observable quantities, we can reveal what principal kinds of space (space-time) are conceivable in the General Theory of Relativity. We show here how to do it, and the result we have obtained.

The operator of projection onto the time line of an observer is the world-vector of his four-dimensional velocity

$$b^{\alpha} = \frac{dx^{\alpha}}{ds}\,, \qquad \alpha = 0, 1, 2, 3, \tag{2.1}$$

with respect to his reference body (the vector is tangential to the world-trajectory of the observer). The theory assumes the observer to be resting with respect to his references. Thus $b^i = 0$ $(i = 1, 2, 3)$, while the rest components of b^{α} are: $b^0 = \frac{1}{\sqrt{g_{00}}}$, $b_0 = g_{0\alpha} b^{\alpha} = \sqrt{g_{00}}$, $b_i = g_{i\alpha} b^{\alpha} = \frac{g_{0i}}{\sqrt{g_{00}}}$. The operator of projection onto the three-dimensional spatial section of the observer is the four-dimensional symmetric tensor

$$h_{\alpha\beta} = -\,g_{\alpha\beta} + b_\alpha b_\beta\,, \tag{2.2}$$

while the properties of the operators are: $b_\alpha b^\alpha = 1$, $h^i_\alpha b^\alpha = 0$, $h^\alpha_i h^k_\alpha = \delta^k_i$.

Thus, any world-vector Q^α has two (observable) chr.inv.-projections, while any 2nd-rank world-tensor $Q^{\alpha\beta}$ has three ones, respectively,

$$b^\alpha Q_\alpha = \frac{Q_0}{\sqrt{g_{00}}}\,, \qquad h^i_\alpha Q^\alpha = Q^i, \tag{2.3}$$

$$b^\alpha b^\beta Q_{\alpha\beta} = \frac{Q_{00}}{g_{00}}\,, \quad h^{i\alpha} b^\beta Q_{\alpha\beta} = \frac{Q^i_0}{\sqrt{g_{00}}}\,, \quad h^i_\alpha h^k_\beta\, Q^{\alpha\beta} = Q^{ik}. \tag{2.4}$$

For instance, projecting a world-coordinate interval dx^α we obtain the interval of the physically observable time

$$d\tau = \sqrt{g_{00}}\,dt + \frac{g_{0i}}{c\sqrt{g_{00}}}\,dx^i, \tag{2.5}$$

and the three-dimensional coordinate interval dx^i. The physically observable velocity is the three-dimensional chr.inv.-vector

$$\mathrm{v}^i = \frac{dx^i}{d\tau}\,, \qquad \mathrm{v}_i \mathrm{v}^i = h_{ik}\, \mathrm{v}^i \mathrm{v}^k = \mathrm{v}^2, \tag{2.6}$$

which along isotropic (light-like) trajectories becomes the physically observable velocity of light c^i, whose square is $c_i c^i = h_{ik}\, c^i c^k = c^2$.

The chr.inv.-metric tensor h_{ik} with the components

$$h_{ik} = -\,g_{ik} + b_i b_k\,, \quad h^{ik} = -\,g^{ik}, \quad h^i_k = -\,g^i_k = \delta^i_k \tag{2.7}$$

is obtained after projecting the fundamental metric tensor $g_{\alpha\beta}$ onto the observer's three-dimensional spatial section. The chr.inv.-operators of differentiation along the line of time and the spatial section

$$\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}\,, \qquad \frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} - \frac{g_{0i}}{g_{00}}\frac{\partial}{\partial x^0}\,, \tag{2.8}$$

are non-commutative

$$\frac{{}^*\partial^2}{\partial x^i\,\partial t} - \frac{{}^*\partial^2}{\partial t\,\partial x^i} = \frac{1}{c^2}\,F_i\,\frac{{}^*\partial}{\partial t}\,, \quad \frac{{}^*\partial^2}{\partial x^i\,\partial x^k} - \frac{{}^*\partial^2}{\partial x^k\,\partial x^i} = \frac{2}{c^2}\,A_{ik}\,\frac{{}^*\partial}{\partial t} \tag{2.9}$$

thus determine the gravitational inertial force F_i acting in the space, and the angular velocity A_{ik} of the space rotation

$$F_i = \frac{1}{1-\frac{\mathrm{w}}{c^2}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right), \tag{2.10}$$

$$A_{ik}=\frac{1}{2}\left(\frac{\partial v_k}{\partial x^i}-\frac{\partial v_i}{\partial x^k}\right)+\frac{1}{2c^2}\left(F_i v_k-F_k v_i\right), \tag{2.11}$$

where $\mathrm{w}=c^2(1-\sqrt{g_{00}})$ is the gravitational potential, while $v_i=-\frac{c\,g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation (its contravariant component $v^i=-c\,g^{0i}\sqrt{g_{00}}$ is determined through $v_i=h_{ik}v^k$ and $v^2=h_{ik}v^iv^k$).

We now express the four-dimensional interval ds through physically observable quantities. We express $g_{\alpha\beta}$ from $h_{\alpha\beta}=-g_{\alpha\beta}+b_\alpha b_\beta$. Thus,

$$ds^2=g_{\alpha\beta}\,dx^\alpha dx^\beta=b_\alpha b_\beta\,dx^\alpha dx^\beta-h_{\alpha\beta}\,dx^\alpha dx^\beta, \tag{2.12}$$

where $b_\alpha dx^\alpha=c\,d\tau$, so the first term is $b_\alpha b_\beta\,dx^\alpha dx^\beta=c^2d\tau^2$. The term $h_{\alpha\beta}\,dx^\alpha dx^\beta$ is the same as the square of the physically observable three-dimensional interval

$$d\sigma^2=h_{ik}\,dx^i dx^k, \tag{2.13}$$

because the theory of chronometric invariants assumes the observer to be resting with respect to his references ($b^i=0$). Thus the four-dimensional interval being expressed through physical observables has the form

$$ds^2=c^2d\tau^2-d\sigma^2. \tag{2.14}$$

According to this formula, three principal kinds of subspace are possible in the space (space-time) of the General Theory of Relativity.

First. The subspace, where

$$ds^2=c^2d\tau^2-d\sigma^2\neq 0\,,\qquad c^2d\tau^2\neq d\sigma^2\neq 0\,, \tag{2.15}$$

is known as the *non-isotropic space*. This is the home of non-isotropic (i.e. nonzero four-dimensional) trajectories and mass-bearing particles, which are both regular subluminal particles and hypothetical superluminal tachyons. Such trajectories lie "within" the light hypercone (the home of subluminal particles), and also "outside" the light hypercone (the home of tachyons).

Second. The subspace, where

$$ds^2=c^2d\tau^2-d\sigma^2=0\,,\qquad c^2d\tau^2=d\sigma^2\neq 0\,, \tag{2.16}$$

is known as the *isotropic space*. This is the home of isotropic (i.e. zero four-dimensional) trajectories. Along such trajectories, the space-time interval is zero, while the interval of the physically observable time and the three-dimensional physically observable interval are nonzero. Isotropic trajectories lie on the surface of the light hypercone, which is the surface of the light speed. Thus the isotropic space hosts particles travelling at the velocity of light. Such particles have zero rest-mass.

They are massless particles, in other words. These are, in particular, photons. For this reason, particles of the isotropic space are also known as massless light-like particles.

These two kinds of space (space-time) are originally well-known commencing in the beginning of the 20th century, once the theory of space-time-matter had been introduced.

We however suggest to consider a third kind of subspace (and particles), which are also theoretically possible in the space (space-time) of the General Theory of Relativity. Consider isotropic (light-like) trajectories in the ultimate case, where, apart from $ds^2 = 0$, they meet even more stricter conditions $c^2 d\tau^2 = 0$ and $d\sigma^2 = 0$, i.e.

$$ds^2 = c^2 d\tau^2 - d\sigma^2 = 0\,, \qquad c^2 d\tau^2 = 0\,, \qquad d\sigma^2 = 0\,. \tag{2.17}$$

This means that not only the space-time interval is zero along such trajectories ($ds^2 = 0$ in any isotropic space). In addition to it, the observable interval of time between any events and all observable three-dimensional lengths are zero therein (being registered by a regular subluminal observer). Therefore, the space wherein such trajectories lie is the ultimate case of the isotropic (light-like) space.

So forth, we go insightfully into the details of the conditions, which characterize a space of this exotic kind. Taking into account the formulae of $d\tau$ (2.5) and $d\sigma$ (2.13), and also the fact that $h_{00} = h_{0i} = 0$, we express the conditions $c^2 d\tau^2 = 0$ and $d\sigma^2 = 0$ in the extended form

$$c\,d\tau = \left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right] c\,dt = 0\,, \qquad dt \neq 0\,, \tag{2.18}$$

$$d\sigma^2 = h_{ik}\, dx^i dx^k = 0\,, \tag{2.19}$$

where $u^i = \frac{dx^i}{dt}$ is the three-dimensional coordinate velocity, which is not a physically observable chr.inv.-quantity.

As is known, the necessary and sufficient condition of full degeneration of a space means zero value of the determinant of the metric tensor, which characterizes the space. For the degenerate three-dimensional physically observable metric $d\sigma^2 = h_{ik}\, dx^i dx^k = 0$ this condition is

$$h = \det \| h_{ik} \| = 0\,. \tag{2.20}$$

On the other hand, as was shown by Zelmanov [10], the determinant $g = \det \| g_{\alpha\beta} \|$ of the fundamental (four-dimensional) metric tensor $g_{\alpha\beta}$ is connected to the determinant of the chr.inv.-metric tensor h_{ik} through the relation

$$g = -\,h\,g_{00}\,. \tag{2.21}$$

Hence degeneration of the three-dimensional metric form $d\sigma^2$, which is characterized by the condition $h=0$, means degeneration of the four-dimensional metric form ds^2, i.e. the condition $g=0$, as well. Therefore a four-dimensional space of the third kind we have herein suggested to consider is a *fully degenerate space*. Respectively, the conditions (2.18) and (2.19) which characterize such a space are the *physical conditions of full degeneration*.

Also, we suggest to refer further to any regular isotropic space as a *particularly degenerate space*. This is because the space-time interval is zero therein, $ds^2=0$, but $c^2d\tau^2\neq 0$ and $d\sigma^2\neq 0$ thus the fundamental metric tensor is not degenerate: $g=\det\|g_{\alpha\beta}\|\neq 0$. In other words, a regular isotropic space is "particularly degenerate".

As has been said above, full degeneration requires not only $ds^2=0$ but also $c^2d\tau^2=0$ and $d\sigma^2=0$. Therefore, we suggest to refer further to any fully degenerate space (space-time) as *zero-space*.

Substituting $h_{ik}=-g_{ik}+b_ib_k=-g_{ik}+\frac{1}{c^2}v_iv_k$ into the second condition (2.19) of those two characterizing a fully degenerate space, then dividing it by dt^2, we obtain the physical conditions of full degeneration, (2.18) and (2.19), in the final form

$$\mathrm{w}+v_iu^i=c^2, \qquad g_{ik}u^iu^k=c^2\left(1-\frac{\mathrm{w}}{c^2}\right)^2, \tag{2.22}$$

where v_iu^i is the scalar product of the linear velocity of the space rotation v_i and the coordinate velocity u^i in the space.

On the basis of the conditions of full degeneration, three subkinds of fully degenerate space (zero-space) are conceivable:

1) If such a space is free of gravitational fields ($\mathrm{w}=0$), the first condition of the conditions of full degeneration (2.22) means $v_iu^i=c^2$, while the second condition of (2.22) becomes $g_{ik}u^iu^k=c^2$. In this particular case, the fully degenerate space rotates with the velocity of light, and all speeds of motion therein are that of light;
2) Once a gravitational field appears in such a space, the space rotation and speeds of motion become slower than light therein according to the conditions of full degeneration (2.22). This is a general case of fully degenerate space;
3) If a fully degenerate space does not rotate ($v_i=0$), the gravitational potential is $\mathrm{w}=c^2$ therein. This means, according to the definition $\mathrm{w}=c^2(1-\sqrt{g_{00}})$ of the potential, that $g_{00}=0$ which is the condition of gravitational collapse. Also, according to the second condition of full degeneration (2.22), the equality $\mathrm{w}=c^2$ means $g_{ik}dx^idx^k=0$. This state, $g_{ik}dx^idx^k=0$, may realize itself

in three cases: a) the three-dimensional coordinate metric g_{ik} degenerates ($\det\|g_{ik}\|=0$); b) all trajectories within the space are shrunk into a point ($dx^i=0$); c) when both these conditions are commonly present in the space. A fully degenerate space of this subkind is collapsed: this is a fully degenerate black hole, in other words. This particular case will be detailed in §4.

About the zero-space metric. As has been said above, all intervals (space-time, time, and spatial ones) are zero in a fully degenerate space from the point of view of an "external" observer located in a regular (non-degenerate) space. The space-time (four-dimensional) interval is invariant, thus its equality to zero remains unchanged in any reference frame. However this is not true about non-invariant quantities, which are the interval of the coordinate time dt and the three-dimensional coordinate interval $g_{ik}\,dx^i dx^k$. As follows from the conditions of full degeneration (2.22), the coordinate quantities can be nonzero in such a space (except in the case of gravitational collapse, where $g_{ik}\,dx^i dx^k=0$). So, we can move from the quantities registered by a regular observer to the coordinate quantities within a fully degenerate space, thus satisfying our curiosity to see what happens therein.

The interval $d\mu^2$ inside a fully degenerate space (i.e. the *zero-space metric*) can be obtained from the second condition of full degeneration (2.22), due to the fact that the three-dimensional coordinate metric g_{ik} does not degenerate. Thus, the zero-space metric has the form

$$d\mu^2 = g_{ik}\,dx^i dx^k = \left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 \neq 0\,, \qquad (2.23)$$

which, due to the first condition of full degeneration is $\mathrm{w}+v_i u^i = c^2$, can be equally expressed as

$$d\mu^2 = g_{ik}\,dx^i dx^k = \frac{v_i v_k\, u^i u^k}{c^2}\, dt^2 \neq 0\,. \qquad (2.24)$$

The zero-space metric manifests that, everywhere in such a space, the following condition

$$g_{ik}\,\overset{*}{u}{}^i\,\overset{*}{u}{}^k = c^2, \qquad (2.25)$$

is true. Here $\overset{*}{u}{}^i = \frac{1}{\sqrt{g_{00}}}\frac{dx^i}{dt} = \frac{{}^*dx^i}{dt}$ is the physical coordinate velocity we introduce through the "starry" derivative with respect to time in analogy to the respective "starry" chr.inv.-derivative (2.8).

According to (2.25), the physical velocities inside a fully degenerate space are always equal to the velocity of light.

The zero-space metric $d\mu^2$ (2.23) is not invariant: $d\mu^2 \neq inv$. This means that the geometry inside a fully degenerate space region is non-Riemannian*. As a result, from the viewpoint of a hypothetical observer located in such a space, the length of the four-dimensional velocity vector does not conserve along its trajectory therein

$$u_\alpha u^\alpha = u_k u^k = g_{ik}\, u^i u^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 \neq const \tag{2.26}$$

but depends on the distribution of the gravitational potential. This fact, in common with the circumstance that the physical velocities therein are equal to the velocity of light, will lead us in §7 to the conclusion that particles, whose home is zero-space, can be associated with virtual photons known due to Quantum Electrodynamics.

§3. The geometric structure of zero-space. So, a regular observer perceives the entire fully degenerate space (zero-space) as a point-like region determined by the observable conditions of full degeneration, which are $d\tau = 0$ and $d\sigma^2 = h_{ik}\, dx^i dx^k = 0$. These conditions mean that he perceives any two events in the zero-space as simultaneous, and also all three-dimensional lengths therein are perceived as zero. Such an observation can be processed at any point of our regular non-degenerate (four-dimensional pseudo-Riemannian) space. This is only possible, if we assume that our space meets the entire zero-space at each point, as it is "stuffed" with the zero-space.

Let us now turn to the geometric interpretation of the conditions of full degeneration. To obtain an illustrated view of the zero-space geometry, we are going to use a *locally geodesic frame of reference*. The fundamental metric tensor within the infinitesimal vicinity of a point in such a frame is

$$\tilde{g}_{\alpha\beta} = g_{\alpha\beta} + \frac{1}{2}\left(\frac{\partial^2 \tilde{g}_{\alpha\beta}}{\partial \tilde{x}^\mu\, \partial \tilde{x}^\nu}\right)(\tilde{x}^\mu - x^\mu)\,(\tilde{x}^\nu - x^\nu) + \dots\,, \tag{3.1}$$

i.e. the numerical values of its components in the vicinity of a point differ from those at this point itself only in the 2nd-order terms or the higher other terms, which can be neglected. Therefore, at any point in a local geodesic frame of reference, the fundamental metric tensor $g_{\alpha\beta}$ is constant (within the 2nd order terms withheld), while the first derivatives of the metric are zero.

*As is known, Riemannian spaces are, by definition, those where: a) the space metric has the square Riemannian form $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$, and b) the metric is invariant $ds^2 = inv$.

It is obvious that a local geodesic frame of reference can be set up within the infinitesimal vicinity of any point in a Riemannian space. As a result, at any point belonging to the local geodesic frame of reference, a flat space can be set up tangential to the Riemannian space so that the local geodesic frame in the Riemannian space is a global geodesic frame in the tangential flat space. Because the fundamental metric tensor is constant in a flat space, the quantities $\tilde{g}_{\alpha\beta}$ converge to those of the tensor $g_{\alpha\beta}$ in the tangential flat space, in the vicinity of any point in the Riemannian space. This means that, in the tangential flat space, we can set up a system of basis vectors $\vec{e}_{(\alpha)}$ tangential to the curved coordinate lines of the Riemannian space. Because the coordinate lines of a Riemannian space are curved (in a general case), and, in the case where the space is non-holonomic*, are not even orthogonal to each other, the lengths of the basis vectors are sometimes substantially different from the unit length.

Consider the world-vector $d\vec{r}$ of an infinitesimal displacement, i.e. $d\vec{r} = (dx^0, dx^1, dx^2, dx^3)$. Then $d\vec{r} = \vec{e}_{(\alpha)} dx^\alpha$, where the components are

$$\left.\begin{array}{ll} \vec{e}_{(0)} = \left(e^0_{(0)}, 0, 0, 0\right), & \vec{e}_{(1)} = \left(0, e^1_{(1)}, 0, 0\right) \\ \vec{e}_{(2)} = \left(0, 0, e^2_{(2)}, 0\right), & \vec{e}_{(3)} = \left(0, 0, 0, e^3_{(3)}\right) \end{array}\right\}. \tag{3.2}$$

The scalar product of the vector $d\vec{r}$ with itself gives $d\vec{r}d\vec{r} = ds^2$. On the other hand, it is $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$. So, we obtain a formula

$$g_{\alpha\beta} = \vec{e}_{(\alpha)}\vec{e}_{(\beta)} = e_{(\alpha)}e_{(\beta)} \cos\left(x^\alpha; x^\beta\right), \tag{3.3}$$

which facilitates our better understanding of the geometric structure of different regions within the space. According to (3.3), therefore,

$$g_{00} = e^2_{(0)}\,, \tag{3.4}$$

where, as is known, g_{00} is included into the formula of the gravitational potential $\mathrm{w} = c^2\left(1 - \sqrt{g_{00}}\right)$. Hence the time basis vector $\vec{e}_{(0)}$ (tangential to the line of time $x^0 = ct$) has the length $e_{(0)} = 1 - \frac{\mathrm{w}}{c^2}$. Thus the lesser the length of $\vec{e}_{(0)}$ is (than 1), the greater the gravitational potential w. In the case of gravitational collapse ($\mathrm{w} = c^2$), the length of the time basis vector $\vec{e}_{(0)}$ becomes zero.

Next, according to (3.3), the quantity g_{0i} is

$$g_{0i} = e_{(0)}e_{(i)} \cos\left(x^0; x^i\right), \tag{3.5}$$

*The non-holonomity of a space (space-time) means that the lines of time are non-orthogonal to the three-dimensional spatial section therein. It manifests as the three-dimensional rotation of the space.

while, on the other hand, $g_{0i} = -\frac{1}{c}\, v_i \left(1 - \frac{\mathrm{w}}{c^2}\right) = -\frac{1}{c}\, v_i\, e_{(0)}$. Hence, the linear velocity of the space rotation, determined as $v_i = -\frac{c\, g_{0i}}{\sqrt{g_{00}}}$, is

$$v_i = -\, c\, e_{(i)} \cos\left(x^0; x^i\right), \tag{3.6}$$

and manifests the angle of inclination of the lines of time towards the spatial section. Then, according to the general formula (3.3), we have

$$g_{ik} = e_{(i)} e_{(k)} \cos\left(x^i; x^k\right), \tag{3.7}$$

hence the chr.inv.-metric tensor $h_{ik} = -\, g_{ik} + \frac{1}{c^2}\, v_i v_k$ has the form

$$h_{ik} = e_{(i)} e_{(k)} \Big[\cos\left(x^0; x^i\right) \cos\left(x^0; x^k\right) - \cos\left(x^i; x^k\right)\Big]. \tag{3.8}$$

From formula (3.6), we see that, from the geometrical viewpoint, v_i is the projection (scalar product) of the spatial basis vector $\vec{e}_{(i)}$ onto the time basis vector $\vec{e}_{(0)}$, multiplied by the velocity of light. If the spatial sections are everywhere orthogonal to the lines of time (giving holonomic space), $\cos\left(x^0; x^i\right) = 0$ and $v_i = 0$. In a non-holonomic space, the spatial sections are not orthogonal to the lines of time, so $\cos\left(x^0; x^i\right) \neq 0$. Generally $\left|\cos\left(x^0; x^i\right)\right| \leqslant 1$, hence the linear velocity of the space rotation v_i can not exceed the velocity of light.

First, consider the geometric structure of the isotropic (light-like) space. It is characterized by the condition $c^2 d\tau^2 = d\sigma^2 \neq 0$. According to this condition, time and regular three-dimensional space meet each other. Geometrically, this means that the time basis vector $\vec{e}_{(0)}$ meets all three spatial basis vectors $\vec{e}_{(i)}$, i.e. time "falls" into space (this fact does not mean that the spatial basis vectors coincide, because the time basis vector is the same for the entire spatial frame). In other words, $\cos\left(x^0; x^k\right) = \pm 1$ everywhere in the isotropic space. At $\cos\left(x^0; x^i\right) = +1$ the time basis vector is co-directed with the spatial ones: $\vec{e}_{(0)} \uparrow\uparrow \vec{e}_{(i)}$. If $\cos\left(x^0; x^i\right) = -1$, the time and spatial basis vectors are oppositely directed: $\vec{e}_{(0)} \uparrow\downarrow \vec{e}_{(i)}$. The condition $\cos\left(x^0; x^k\right) = \pm 1$ can be expressed through the gravitational potential $\mathrm{w} = c^2 \left(1 - \sqrt{g_{00}}\right)$, because, in a general case, $e_{(0)} = \sqrt{g_{00}}$ (3.4). Finally, we obtain the geometric conditions which characterize the isotropic space. They are

$$\cos\left(x^0; x^k\right) = \pm 1\,, \qquad e_{(i)} = e_{(0)} = \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}\,, \tag{3.9}$$

and, hence,

$$v_i = \mp\, c e_{(i)} = \mp \sqrt{g_{00}}\; c_i = \mp \left(1 - \frac{\mathrm{w}}{c^2}\right) c_i\,, \tag{3.10}$$

$$h_{ik} = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 \left[\,1 - \cos\left(x^i; x^k\right)\right], \tag{3.11}$$

where c^i is the chronometrically invariant three-dimensional vector of the physically observable velocity of light, $c_i c^i = h_{ik}\, c^i c^k = c^2$.

According to the obtained formula (3.10), we conclude, as well as it was primarily concluded by one of us in a previous study [12]:

> The isotropic space rotates at each point with a linear velocity, which is basically equal, to the velocity of light, and is slowing down in the presence of the gravitational potential.

The isotropic space exists at any point in the four-dimensional regular space as a light hypercone — a hypersurface whose metric is

$$g_{\alpha\beta}\, dx^{\alpha} dx^{\beta} = 0\,, \tag{3.12}$$

or, in the extended form,

$$\left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 - 2\left(1-\frac{\mathrm{w}}{c^2}\right) v_i\, dx^i dt + g_{ik}\, dx^i dx^k = 0\,, \tag{3.13}$$

according to the formulae of the gravitational potential $\mathrm{w} = c^2\,(1-\sqrt{g_{00}}\,)$ and the linear velocity of the space rotation $v_i = -\frac{c\, g_{0i}}{\sqrt{g_{00}}}$.

This is a subspace of the four-dimensional space which hosts massless (light-like) particles travelling at the velocity of light. Because the space-time interval in such a region is zero, all four-dimensional directions inside it are equal (in other words, they are isotropic). Therefore this subspace is commonly referred to as the *isotropic hypercone.*

Let us now turn to the geometric structure of the zero-space. Because w and v_i, being written in the basis form, are $\mathrm{w} = c^2(1-e_{(0)})$ and $v_i = -\,c\, e_{(i)} \cos\,(x^0; x^i)$, the condition of full degeneration $\mathrm{w} + v_i u^i = c^2$ can be written in the basis form as well

$$c\, e_{(0)} = -\, e_{(i)}\, u^i \cos\,(x^0;\, x^i)\,. \tag{3.14}$$

This formula can be regarded as the *geometric condition of full degeneration.*

Because the four-dimensional metric is also equal to zero in the zero-space, such a space exists at any point of the isotropic (light) hypercone as a fully degenerate subspace of it. Such a *fully degenerate isotropic hypercone* is described by a somewhat different equation

$$\left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 - g_{ik}\, dx^i dx^k = 0\,, \tag{3.15}$$

or, due to the zero-space metric, which can equally be presented as (2.23) and (2.24), by the equation

$$\frac{v_i v_k\, u^i u^k}{c^2}\, dt^2 - g_{ik}\, dx^i dx^k = 0\,. \tag{3.16}$$

The difference between such a fully degenerate isotropic hypercone and the regular isotropic (light) hypercone is that the first satisfies the condition of full degeneration $\mathrm{w} + v_i u^i = c^2$. Because v_i is expressed in both cases in the same form (3.10), we arrive at the conclusion:

> The fully degenerate isotropic hypercone is a cone of light-speed rotation as well as the regular isotropic hypercone. In other words, the zero-space rotates at its each point with a linear velocity equal to the velocity of light. Its rotation becomes slower than light in the presence of the gravitational potential.

Finally, we conclude that the regular isotropic (light) hypercone contains the degenerate isotropic hypercone, which is the entire zero-space, as a subspace embedded into it at its each point. This is a clear illustration of the fractal structure of the world presented here as a system of the isotropic cones found inside each other.

§4. Gravitational collapse in a zero-space region. Fully degenerate black holes. As is known, *gravitational collapsar* or *black hole* is a local region of space (space-time), wherein the condition $g_{00} = 0$ is true. Because the gravitational potential is defined as $\mathrm{w} = c^2 (1 - \sqrt{g_{00}})$, the gravitational collapse condition $g_{00} = 0$ means that the gravitational potential is $\mathrm{w} = c^2$ in the region. We are going to consider how this condition can be realized in zero-space.

The first condition of full degeneration (2.22) is $\mathrm{w} + v_i u^i = c^2$. According to the condition, if $v_i u^i = 0$ in a local zero-space region, the gravitational potential is $\mathrm{w} = c^2$ therein. This means that, in the case of $v_i u^i = 0$, the gravitational potential is strong enough to bring the local region of zero-space to gravitational collapse. We suggest to refer to such a region as a *fully degenerate gravitational collapsar* or, equivalently, as a *fully degenerate black hole*.

The second condition of full degeneration becomes $g_{ik}\, dx^i dx^k = 0$ in this case. Together with the previous, this means that three physical and geometric conditions are realized in fully degenerate black holes

$$\mathrm{w} = c^2, \qquad v_i u^i = 0\,, \qquad g_{ik}\, dx^i dx^k = 0\,, \tag{4.1}$$

whose physical meaning is as follows:

1) The gravitational potential inside fully degenerate black holes is strong enough to stop the regular light-speed rotation of the local region of zero-space, i.e.

$$v_i = \mp\, c e_{(i)} = \mp \sqrt{g_{00}}\; c_i = \mp \left(1 - \frac{\mathrm{w}}{c^2}\right) c_i = 0\,; \tag{4.2}$$

2) In this case, the time basis vector $\vec{e}_{(0)}$ has zero length (intervals of time are zero inside fully degenerate black holes)

$$e_{(0)} = \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2} = 0\,; \tag{4.3}$$

3) In any case of zero-space, the condition $\cos(x^0; x^k) = \pm 1$ is true: the time basis vector $\vec{e}_{(0)}$ meets all three spatial basis vectors $\vec{e}_{(i)}$ (time "falls" into space). Therefore, the previous condition $e_{(0)} = 0$ means that all three three-dimensional (spatial) basis vectors $\vec{e}_{(i)}$ have zero length inside fully degenerate black holes as well, i.e.

$$e_{(i)} = e_{(0)} = \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2} = 0\,; \tag{4.4}$$

4) The condition $e_{(i)} = 0$ means that the entire three-dimensional space inside fully degenerate black holes is shrunk into a point (all three-dimensional coordinate intervals are $dx^i = 0$). Hence, the third condition $g_{ik}\,dx^i dx^k = 0$ of the conditions inside fully degenerate black holes (4.1) is due to $dx^i = 0$, while the three-dimensional coordinate metric is not degenerate therein

$$\det\|\,g_{ik}\| \neq 0\,. \tag{4.5}$$

Hence fully degenerate black holes are point-like objects, which keep light stored inside themselves due to their own ultimately strong gravitation. In other words, they are "absolute black holes" of all gravitational collapsars theoretically conceivable due to the General Theory of Relativity.

§5. Zero-space: the gate for teleporting photons. As we mentioned above, a regular observer may connect to the entire fully degenerate space (zero-space) at any point or local region of the regular space once the observable conditions of full degeneration, which are $d\tau = 0$ and $d\sigma^2 = h_{ik}\,dx^i dx^k = 0$, are realized therein. The physical meaning of the first condition $d\tau = 0$ is that the regular observer perceives any two events in the zero-space region as simultaneous, at whatever distance from them they are located. We will further refer to such a way of instantaneous transfer of information as the *long-range action*. A process in which a particle (a mediator of the interaction) may realize the long-range action will be referred to as *teleportation*.

Therefore, the first condition of full degeneration $d\tau = 0$, which can also be extended due to the definition of $d\tau$ (2.5) as

$$d\tau = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt - \frac{1}{c^2}\, v_i\, dx^i = 0\,, \tag{5.1}$$

thus expressed in the form $\mathrm{w} + v_i u^i = c^2$ (2.22), has also the physical meaning of the *teleportation condition*.

Mediators of the long-range action are particles, which are a sort of photons. This is because, as was detailed in page 8, the physical conditions inside a zero-space region are the ultimate case of the conditions of the regular isotropic (light-like) space, which is the home of photons. In other words, the long-range action is transferred by special "fully degenerate photons", which exist under the physical conditions of full degeneration. Such particles, what they are and how they seem from the point of view of a regular observer, will be discussed in §6–§8.

Once a photon has entered into a local zero-space region at one location of our regular space, it can be instantly connected to another photon which has simultaneously entered into another zero-space "gate" at another distant location. From the point of view of a regular "external" observer, such a connexion is realized instantly. However, inside the zero-space itself, fully degenerate photons transfer interaction between these two locations with the velocity of light (see comments to formula 2.25 in page 10, for details).

Thus, we conclude that instant transfer of information is naturally permitted in the framework of the General Theory of Relativity, despite the real speeds of particles not exceeding the velocity of light. This is merely a "space-time trick", which may only be due to the space-time geometry and topology: we only see that the information is transferred instantaneously, while it is transferred by not-faster-than-light particles travelling in another space which seems to us, the "external" observers, as that wherein all intervals of time and all three-dimensional spatial intervals are zero.

Until this day, teleportation has had an explanation given only by Quantum Mechanics [13]. It was previously achieved only in the strict quantum fashion — quantum teleportation of photons in 1998 [14] and of atoms in 2004 [15, 16]. Now the situation changes: with our theory we can find physical conditions for teleportation of photons in a non-quantum way, which is not due to the probabilistic laws of Quantum Mechanics but according to the exact (non-quantum) laws of the General Theory of Relativity following the space-time geometry. We therefore suggest to refer to this fashion as *non-quantum teleportation*.

§6. Zero-particles: particles which inhabit zero-space. As is obvious, the fully degenerate space can only host such particles for which the physical conditions of full degeneration are true. The properties of such particles will now be under focus. We will start this consideration from the regular (non-degenerate) particles, then apply the physical conditions of full degeneration, thus determining the characteristics of the particles hosted by the fully degenerate space (zero-space).

According to the General Theory of Relativity [17], any mass-bearing particle is characterized by the four-dimensional vector of momentum

$$P^{\alpha} = m_0 \frac{dx^{\alpha}}{ds}\,, \tag{6.1}$$

where m_0 is the rest-mass characterizing the particle. In the framework of de Broglie's wave-particle duality, we can represent the same mass-bearing particle as a wave characterized by the four-dimensional wave vector

$$K^{\alpha} = \frac{\omega_0}{c} \frac{dx^{\alpha}}{ds}\,, \tag{6.2}$$

while ω_0 is the rest-frequency of the de Broglie wave. The square of the momentum vector P^{α} and the wave vector K^{α} along the trajectory of each single mass-bearing particle is constant, which is nonzero

$$P_{\alpha}P^{\alpha} = g_{\alpha\beta}\, P^{\alpha}P^{\beta} = m_0^2 = const \neq 0\,, \tag{6.3}$$

$$K_{\alpha}K^{\alpha} = g_{\alpha\beta}\, K^{\alpha}K^{\beta} = \frac{\omega_0^2}{c^2} = const \neq 0\,, \tag{6.4}$$

i.e. P^{α} and K^{α} are non-isotropic vectors in this case.

As is seen, the space-time interval ds is applied as the derivation parameter for mass-bearing particles. It works, because such particles travel along non-isotropic trajectories, where, as is known, $ds \neq 0$. Massless (light-like) particles inhabit the isotropic space. They travel along isotropic trajectories, where $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$ and $c^2 d\tau^2 = d\sigma^2 \neq 0$. The space-time interval is $ds = 0$ therein, and thus cannot be applied as the derivation parameter. Zelmanov [10] had removed this problem by suggesting the observable three-dimensional observable interval, which is $d\sigma \neq 0$ along isotropic trajectories. Moreover, $d\sigma$ and $d\tau$ are chronometric invariants: they are invariant along the three-dimensional spatial section of the observer. Therefore they can be used as derivation parameters along both isotropic and non-isotropic trajectories, in the framework of the chronometrically invariant formalism.

Since ds^2 in the chr.inv.-form (2.14) can be expressed through the physically observable chr.inv.-velocity v^i (2.6) as

$$ds^2 = c^2 d\tau^2 - d\sigma^2 = c^2 d\tau^2 \left(1 - \frac{\mathrm{v}^2}{c^2}\right), \tag{6.5}$$

we can write down the regular formulae of P^α (6.1) and K^α (6.2) as

$$P^\alpha = m\,\frac{dx^\alpha}{d\sigma} = \frac{m}{c}\,\frac{dx^\alpha}{d\tau}\,, \tag{6.6}$$

$$K^\alpha = \frac{\omega}{c}\,\frac{dx^\alpha}{d\sigma} = \frac{k}{c}\,\frac{dx^\alpha}{d\tau}\,, \tag{6.7}$$

where m is the relativistic mass (derived for massless particles through their relativistic energy $E = mc^2$), ω is the relativistic frequency, while $k = \frac{\omega}{c}$ is the wave number.

In the case of massless particles (isotropic trajectories), the square of the momentum vector P^α and the wave vector K^α is zero

$$P_\alpha P^\alpha = g_{\alpha\beta}\,P^\alpha P^\beta = \frac{m^2}{c^2}\frac{g_{\alpha\beta}\,dx^\alpha dx^\beta}{d\sigma^2} = \frac{m^2}{c^2}\frac{ds^2}{d\sigma^2} = 0\,, \tag{6.8}$$

$$K_\alpha K^\alpha = g_{\alpha\beta}\,K^\alpha K^\beta = \frac{\omega^2}{c^2}\frac{g_{\alpha\beta}\,dx^\alpha dx^\beta}{d\sigma^2} = \frac{\omega^2}{c^2}\frac{ds^2}{d\sigma^2} = 0\,, \tag{6.9}$$

i.e. P^α and K^α are isotropic vectors in this case.

Calculation of the contravariant components of P^α and K^α gives

$$P^0 = m\,\frac{dt}{d\tau}\,, \qquad P^i = \frac{m}{c}\frac{dx^i}{d\tau} = \frac{1}{c}\,m\mathrm{v}^i, \tag{6.10}$$

$$K^0 = k\,\frac{dt}{d\tau}\,, \qquad K^i = \frac{k}{c}\frac{dx^i}{d\tau} = \frac{1}{c}\,k\mathrm{v}^i, \tag{6.11}$$

where $m\mathrm{v}^i$ is the three-dimensional chr.inv.-momentum vector, while $k\mathrm{v}^i$ is the three-dimensional chr.inv.-wave vector.

The function $\frac{dt}{d\tau}$ can be obtained from the equation of the square of the four-dimensional velocity, which is $g_{\alpha\beta}\,u^\alpha u^\beta = +1$ for subluminal velocities, $g_{\alpha\beta}\,u^\alpha u^\beta = 0$ for the velocity of light, and $g_{\alpha\beta}\,u^\alpha u^\beta = -1$ for superluminal velocities. Extending $g_{\alpha\beta}\,u^\alpha u^\beta$ to component notation, then substituting the definitions of h_{ik}, v_i, v^i into each of these three formulae, we arrive at the same quadratic equation

$$\left(\frac{dt}{d\tau}\right)^2 - \frac{2\,v_i\mathrm{v}^i}{c^2\left(1-\frac{\mathrm{w}}{c^2}\right)}\frac{dt}{d\tau} + \frac{1}{\left(1-\frac{\mathrm{w}}{c^2}\right)^2}\left(\frac{1}{c^4}\,v_i v_k \mathrm{v}^i\mathrm{v}^k - 1\right) = 0\,, \tag{6.12}$$

which solves (to within positive roots) as

$$\frac{dt}{d\tau} = \frac{1}{1-\frac{\mathrm{w}}{c^2}}\left(\frac{1}{c^2}\,v_i\mathrm{v}^i + 1\right). \tag{6.13}$$

With this solution, we obtain the covariant components P_i and K_i, then — the chr.inv.-projections of P^α and K^α onto the line of time

$$P_i = -\frac{m}{c}\left(\mathrm{v}_i + v_i\right), \qquad \frac{P_0}{\sqrt{g_{00}}} = m\,, \tag{6.14}$$

$$K_i = -\frac{k}{c}\left(\mathrm{v}_i + v_i\right), \qquad \frac{K_0}{\sqrt{g_{00}}} = k\,. \tag{6.15}$$

According to the chronometrically invariant formalism (see formula (2.3) for detail), any world-vector Q^α has two physically observable projections: $\frac{Q_0}{\sqrt{g_{00}}}$ and Q^i. Hence, the physical observables are

1) the relativistic mass m,

2) the three-dimensional momentum $m\mathrm{v}^i$,

which are represented, in the framework of de Broglie's wave-particle duality, respectively by

1) the wave number $k = \frac{\omega}{c}$,

2) the three-dimensional wave vector $k\mathrm{v}^i$.

In the case of massless particles (isotropic trajectories), v^i is equal to the physically observable chr.inv.-velocity of light c^i.

Now, we apply the physical conditions of full degeneration to the obtained formulae, thus considering the particles hosted by the fully degenerate space.

Using the definition of $d\tau$ (2.5), we obtain the relation between the coordinate velocity u^i and the physical observable velocity v^i

$$\mathrm{v}^i = \frac{u^i}{1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)}\,, \tag{6.16}$$

which takes the first condition of full degeneration $\mathrm{w} + v_i u^i = c^2$ (2.22) into account. Thus, we express ds^2 in the form

$$ds^2 = c^2 d\tau^2\left(1 - \frac{\mathrm{v}^2}{c^2}\right) = c^2 dt^2\left\{\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)\right]^2 - \frac{u^2}{c^2}\right\}, \tag{6.17}$$

containing the first condition of full degeneration as well. Hence, the four-dimensional vector of momentum can be expressed in the form

$$P^\alpha = m_0\,\frac{dx^\alpha}{ds} = \frac{M}{c}\frac{dx^\alpha}{dt}\,, \tag{6.18}$$

$$M = \frac{m_0}{\sqrt{\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)\right]^2 - \frac{u^2}{c^2}}}\,. \tag{6.19}$$

Such a mass M depends not only on the three-dimensional velocity of the particle with respect to the observer, but also on the gravitational potential w, and on the linear velocity of the rotation v_i of space at the point of observation.

Substituting, into the formula of M, the quantity $\mathrm{v}^2 = h_{ik}\mathrm{v}^i\mathrm{v}^k$ derived from (6.16), and m_0 expressed through m, we arrive at the relation between the relativistic mass m and the mass M

$$M = \frac{m}{1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)}\,. \tag{6.20}$$

From the obtained formula we see that M, under the first condition of full degeneration $\mathrm{w} + v_i u^i = c^2$, becomes a ratio between two quantities, each one is equal to zero, but the ratio itself is not zero: $M \neq 0$. This fact is not a surprise. The same is true for the relativistic mass m in the case of $\mathrm{v} = c$, which is the case of massless (light-like) particles. Once there $m_0 = 0$ in the numerator, and the relativistic square-root term is zero in the denominator (due to $\mathrm{v} = c$), the ratio of these quantities is still $m \neq 0$.

In analogy to the momentum vector P^α, we can represent the wave vector K^α is the form

$$K^\alpha = \frac{\omega_0}{c}\frac{dx^\alpha}{ds} = \frac{\Omega}{c^2}\frac{dx^\alpha}{dt}\,, \tag{6.21}$$

$$\Omega = \frac{\omega_0}{\sqrt{\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)\right]^2 - \frac{u^2}{c^2}}} = \frac{\omega}{1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)}\,, \tag{6.22}$$

which also takes the first condition of full degeneration into account.

It is easy to obtain that the components of the momentum vector in the fully degenerate space (zero-space) are

$$P^0 = M \neq 0\,, \qquad P^i = \frac{1}{c}\,M u^i \neq 0\,, \qquad P_i = -\frac{1}{c}\,M u_i \neq 0\,, \tag{6.23}$$

$$\frac{P_0}{\sqrt{g_{00}}} = M\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right] = m = 0\,, \tag{6.24}$$

while the components of the wave vector are

$$K^0 = \frac{\Omega}{c} \neq 0\,, \qquad K^i = \frac{1}{c^2}\,\Omega u^i \neq 0\,, \qquad K_i = -\frac{1}{c^2}\,\Omega u_i \neq 0\,, \tag{6.25}$$

$$\frac{K_0}{\sqrt{g_{00}}} = \frac{\Omega}{c}\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right] = \frac{\omega}{c} = 0\,. \tag{6.26}$$

As is seen, the physically observable quantities $\frac{P_0}{\sqrt{g_{00}}}$ (6.24) and $\frac{K_0}{\sqrt{g_{00}}}$ (6.26), which are the projections of the world-vectors P^α and K^α onto the line of time, become zero under the first condition of full degeneration $\mathrm{w} + v_i u^i = c^2$. This is because, despite the quantities M and Ω being nonzero, their multiplier in the brackets becomes zero under the condition. This means, according to the obtained formulae (6.24) and (6.26), that the relativistic mass m and the relativistic frequency ω (which corresponds to the relativistic mass within de Broglie's wave-particle duality) are zero in the fully degenerate space.

As a result, we can conclude something about the physically observable characteristics of the particles hosted by the fully degenerate space (zero-space):

1) Such fully degenerate particles bear zero relativistic mass ($m = 0$) and zero relativistic de Broglie frequency ($\omega = 0$);
2) They also bear zero rest-mass ($m_0 = 0$). This follows from the fact that the physical conditions inside a zero-space region are the ultimate case of the conditions of the regular isotropic (light-like) space, which is the home of photons (see page 8 for detail).

Therefore, the particles hosted by the fully degenerate space (zero-space) are the ultimate case of photons, which exist under the conditions of full degeneration. They are "fully degenerate photons", in other words. Since not only their rest-mass m_0, but also the relativistic mass m and frequency ω are zero, we suggest to refer further to such fully degenerate particles as *zero-particles.*

§7. Insight into zero-space: zero-particles as virtual photons. As is well-known, the Feynman diagrams are a graphical description of interactions between elementary particles. The diagrams show that the actual carriers of the interactions are virtual particles. In other words, almost all physical processes rely on the emission and the absorption of virtual particles (e.g. virtual photons) by real particles of our world.

Hence, to give a geometric interpretation of the Feynman diagrams in the space-time of the General Theory of Relativity, we only need a formal definition for virtual particles. Here is how to do it.

According to Quantum Electrodynamics, virtual particles are those for which, contrary to regular ones, the regular relation between energy and momentum

$$E^2 - c^2 p^2 = E_0^2 \,, \tag{7.1}$$

where $E = mc^2$, $p^2 = m^2 \mathrm{v}^2$, $E_0 = m_0 c^2$, is not true. In other words, for virtual particles,

$$E^2 - c^2p^2 \neq E_0^2\,. \tag{7.2}$$

In a pseudo-Riemannian space, the regular relation (7.1) is true. It follows from the condition $P_\alpha P^\alpha = m_0^2 = const \neq 0$ for mass-bearing particles (non-isotropic trajectories), and from the condition $P_\alpha P^\alpha = 0$ for massless particles (isotropic trajectories). Substituting the respective components of the momentum vector P^α, we obtain the regular relation, in the chr.inv.-form, for mass-bearing particles,

$$E^2 - c^2m^2\,\mathrm{v}_i\mathrm{v}^i = E_0^2\,, \tag{7.3}$$

and that for massless ones, $E^2 - c^2m^2\,\mathrm{v}_i\mathrm{v}^i = 0$, that is the same as

$$h_{ik}\,\mathrm{v}^i\mathrm{v}^k = c^2. \tag{7.4}$$

But this is not true in the fully degenerate space (zero-space). This is because the zero-space metric $d\mu^2$ (2.23) is not invariant: $d\mu^2 \neq inv$. As a result, from the viewpoint of a hypothetical observer who is located therein, a degenerate four-velocity vector being transferred in parallel to itself does not conserve its length: $u_\alpha u^\alpha \neq const$ (2.26). Therefore, the regular relation between energy and momentum $E^2 - c^2p^2 = const$ (7.1) is not applicable to zero-particles, but another relation, which is a sort of $E^2 - c^2p^2 \neq const$ (7.2), is true. Because the latter is the main property of virtual particles, we arrive at the conclusion:

> Zero-particles may play a rôle of virtual particles, which, according to Quantum Electrodynamics, are material carriers of interaction between regular particles of our world. If so, the entire zero-space is an "exchange buffer" in whose capacity zero-particles transfer interactions between regular mass-bearing and massless particles of our world.

As has been shown on page 22, zero-particles are fully degenerate photons. They can also exist in a collapsed region of zero-space, wherein the condition of gravitational collapse is true (see §4). Hence, virtual particles of two kinds can be presupposed:

1) Virtual photons — regular fully degenerate photons;
2) Virtual collapsars — fully degenerate photons, which are hosted by the collapsed regions of zero-space.

All that we have suggested here is for yet the sole explanation of virtual particles and virtual interactions given by the geometric methods of the General Theory of Relativity, and according to the geometric structure of the four-dimensional space (space-time).

§8. Zero-particles from the point of view of a regular observer: standing light waves. The following important question rises: if zero-particles bear zero rest-mass and zero relativistic mass, how can they be perceived by a regular observer like us who are located in the regular (non-degenerate) space? To answer this question, we now consider zero-particles in framework of the geometric optics approximation.

As is known [17], the four-dimensional wave vector of massless particles in the geometric optics approximation is

$$K_\alpha = \frac{\partial\psi}{\partial x^\alpha}\,, \tag{8.1}$$

where ψ is the wave phase (eikonal). In analogy to K_α, we suggest to introduce the four-dimensional vector of momentum

$$P_\alpha = \frac{\hbar}{c}\,\frac{\partial\psi}{\partial x^\alpha}\,, \tag{8.2}$$

where $\hbar$ is Planck's constant, while the coefficient $\frac{\hbar}{c}$ equates the dimensions of both parts of the equation. We obtain the physically observable projections of these world-vectors onto the line of time

$$\frac{K_0}{\sqrt{g_{00}}} = \frac{1}{c}\,\frac{{}^*\partial\psi}{\partial t}\,, \qquad \frac{P_0}{\sqrt{g_{00}}} = \frac{\hbar}{c^2}\,\frac{{}^*\partial\psi}{\partial t}\,. \tag{8.3}$$

Equating these to the respective formulae obtained in §6, we obtain that the relativistic frequency and mass are formulated, in the framework of the geometric optics approximation, as

$$\omega = \frac{{}^*\partial\psi}{\partial t}\,, \qquad m = \frac{\hbar}{c^2}\,\frac{{}^*\partial\psi}{\partial t}\,, \tag{8.4}$$

and, respectively, the generalized frequency and mass are

$$\Omega = \frac{1}{1-\frac{1}{c^2}\left(\mathrm{w}+v_i u^i\right)}\,\frac{{}^*\partial\psi}{\partial t}\,, \quad M = \frac{\hbar}{c^2\bigl[1-\frac{1}{c^2}\left(\mathrm{w}+v_i u^i\right)\bigr]}\,\frac{{}^*\partial\psi}{\partial t}\,. \tag{8.5}$$

Thus, we have a possibility of obtaining the respective formulae for the energy and momentum of a particle, expressed through its wave phase in the framework of the geometric optics approximation. In the fully degenerate space (zero-space), the relativistic mass, momentum, frequency, and energy are zero. However, the generalized mass M, momentum Mu^i, frequency Ω, and energy E are nonzero therein (see §6 for detail). As a result of (8.5), we obtain the formulae

$$E = \hbar\,\Omega = Mc^2 = \frac{\hbar}{1-\frac{1}{c^2}\left(\mathrm{w}+v_i u^i\right)}\,\frac{{}^*\partial\psi}{\partial t}\,, \tag{8.6}$$

$$Mu^i = -\,\hbar\, h^{ik}\,\frac{{}^*\partial\psi}{\partial x^k}\,, \tag{8.7}$$

which, in the regular (non-degenerate) space, transform into

$$E = \hbar\omega = mc^2 = \hbar\,\frac{{}^*\partial\psi}{\partial t}\,, \qquad m\mathrm{v}^i = -\,\hbar\, h^{ik}\,\frac{{}^*\partial\psi}{\partial x^k}\,. \tag{8.8}$$

As is known [17], the condition $K_\alpha K^\alpha = 0$, which is specific to massless particles (isotropic trajectories), has the form

$$g^{\alpha\beta}\,\frac{\partial\psi}{\partial x^\alpha}\,\frac{\partial\psi}{\partial x^\beta} = 0\,, \tag{8.9}$$

which is the basic equation of geometric optics (the eikonal equation). After formulating the regular differential operators through the chr.inv.-differential operators (2.8), and taking into account the main property $g_{\alpha\sigma}\,g^{\beta\sigma} = \delta_\alpha^\beta$ of the tensor $g_{\alpha\beta}$, which gives $g^{00} = \frac{1}{g_{00}}\left(1 - \frac{1}{c^2}\,v_i\,v^i\right)$, we arrive at the chr.inv.-eikonal equation for massless particles

$$\frac{1}{c^2}\left(\frac{{}^*\partial\psi}{\partial t}\right)^2 - h^{ik}\,\frac{{}^*\partial\psi}{\partial x^i}\,\frac{{}^*\partial\psi}{\partial x^k} = 0\,. \tag{8.10}$$

In the same way, proceeding from the condition $P_\alpha P^\alpha = m_0^2$ characterizing mass-bearing particles (non-isotropic trajectories), we obtain the chr.inv.-eikonal equation for mass-bearing particles

$$\frac{1}{c^2}\left(\frac{{}^*\partial\psi}{\partial t}\right)^2 - h^{ik}\,\frac{{}^*\partial\psi}{\partial x^i}\,\frac{{}^*\partial\psi}{\partial x^k} = \frac{m_0^2 c^2}{\hbar^2}\,, \tag{8.11}$$

which when $m_0 = 0$ becomes the same as the former one.

To obtain the chr.inv.-eikonal equation for zero-particles, we apply the conditions $m_0 = 0$, $m = 0$, $\omega = 0$, and $P_\alpha P^\alpha = 0$, which characterize the fully degenerate space (zero-space). After some algebra we obtain the chr.inv.-eikonal equation for zero-particles

$$h^{ik}\,\frac{{}^*\partial\psi}{\partial x^i}\,\frac{{}^*\partial\psi}{\partial x^k} = 0\,. \tag{8.12}$$

As is seen, this is a standing wave equation. This fact, and also that zero-particles are the ultimate case of light-like particles (see page 22 for details), allows us to conclude how zero-particles could be registered experimentally:

> Zero-particles should seem from the point of view of a regular observer as *standing light waves* — the waves of stopped light, in other words. So, the entire zero-space should appear filled with a system of standing light waves (light-like holograms).

§9. Conclusion. What is frozen light? So, the geometric structure of the four-dimensional space (space-time) of the General Theory of Relativity manifests the possibility of the ultimate case of photons, for which not only the rest-mass is zero (as for regular photons), but also the relativistic mass is zero. We therefore refer to them as *zero-particles*. Such particles are hosted by a space with fully degenerate metric, which is the ultimate case of the light-like (particularly degenerate) space. They are fully degenerate photons, in other words.

Zero-particles can be hosted by both regular regions and collapsed regions of the fully degenerate space. In the latter case, they exist under the condition of gravitational collapse (see §4).

The fully degenerate space looks like a local volume, wherein all observable intervals of time and all three-dimensional observable intervals are zero. Once a photon has entered into such a zero-space "gate" at one location of our regular space, it can be instantly connected to another photon which has entered into a similar "gate" at another location. This is a way for *non-quantum teleportation* of photons (see §5).

Also, the regular relation between energy and momentum is not true for zero-particles. This means that zero-particles may play a rôle of virtual particles, which are material carriers of interaction between regular particles of our world (see §7).

From the point of view of a regular observer, zero-particles should appear as standing light waves — the waves of stopped light (see §8). The latter meets that which has been registered in the frozen light experiment: there, a light beam being stopped is "stored" in atomic vapor, and remains invisible to the observer until that moment of time when it is set free again in its regularly "travelling state". (See Introduction and the original reports about the experiments referred therein.)

This means that the frozen light experiment pioneered at Harvard by Lene Hau is an experimental "foreword" to the discovery of zero-particles and, hence, a way for non-quantum teleportation.

With these we can mean frozen light as a new state of matter, which differs from the others (solid, gas, liquid, plasma).

Submitted on November 17, 2010

1. Hau L. V., Harris S. E., Dutton Z., Behroozi C. H. Light speed reduction to 17 metres per second in an ultracold atomic gas. *Nature*, 18 February 1999, vol. 397, 594–598.

2. Liu C., Dutton Z., Behroozi C. H., Hau L. V. Observation of coherent optical information storage in an atomic medium using halted light pulses. *Nature*, 25 January 2001, vol. 409, 490–493.

3. Dutton Z., Budde M., Slowe C., Hau L. V. Observation of quantum shock waves created with ultra-compressed slow light pulses in a Bose-Einstein condensate. *Science*, 27 July 2001, vol. 293, 663–668.
4. Hau L. V. Frozen light. *Scientific American*, 17 July 2001, vol. 285, no. 1, 52–59.
5. Hau L. V. Frozen light. *Scientific American Special Edition "The Edge of Physics"*, 31 May 2003, 44–51.
6. Zhang R., Garner S. R., Hau L. V. Creation of long-term coherent optical memory via controlled nonlinear interactions in Bose-Einstein condensates. *Physical Review Letters*, 4 December 2009, vol. 103, 233602.
7. Phillips D. F., Fleischhauer A., Mair A., Walsworth R. L., Lukin M. D. Storage of light in atomic vapor. *Physical Review Letters*, 29 January 2001, vol. 86, no. 5, 783–786.
8. Turukhin A. V., Sudarshanam V. S., Shahriar M. S., Musser J. A., Ham B. S., Hemmer P. R. Observation of ultraslow and stored light pulses in a solid. *Physical Review Letters*, 14 January 2002, vol. 88, no. 2, 023602.
9. Rabounski D. and Borissova L. Particles Here and Beyond the Mirror. 2nd edition, Svenska fysikarkivet, Stockholm, 2009 (first published by Editorial URSS Publishers, Moscow, 2001).
10. Zelmanov A. L. Chronometric Invariants: On Deformations and the Curvature of Accompanying Space. Translated from the preprint of 1944, American Research Press, Rehoboth (NM), 2006.
11. Zelmanov A. L. Chronometric invariants and accompanying frames of reference in the General Theory of Relativity. *Soviet Physics Doklady*, 1956, vol. 1, 227–230 (translated from *Doklady Academii Nauk USSR*, 1956, vol. 107, no. 6, 815–818).
12. Rabounski D. On the speed of rotation of isotropic space: insight into the redshift problem. *The Abraham Zelmanov Journal*, 2009, vol. 2, 208–223.
13. Bennett C. H., Brassard G., Crepeau C., Jozsa R., Peres A., and Wootters W. K. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Physical Review Letters*, 1993, vol. 70, 1895–1899.
14. Boschi D., Branca S., De Martini F., Hardy L., and Popescu S. Experimental realization of teleporting an unknown pure quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Physical Review Letters*, 1998, vol. 80, 1121–1125.
15. Riebe M., Häffner H., Roos C. F., Hänsel W., Benhelm J., Lancaster G. P. T., Korber T. W., Becher C., Schmidt-Kaler F., James D. F. V., and Blatt R. Deterministic quantum teleportation with atoms. *Nature*, 17 June 2004, vol. 429, 734–737.
16. Barrett M. D., Chiaverini J., Schaetz T., Britton J., Itano W. M., Jost J. D., Knill E., Langer C., Leibfried D., Ozeri R., and Wineland D. J. Deterministic quantum teleportation of atomic qubits. *Nature*, 17 June 2004, vol. 429, 737–739.
17. Landau L. D. and Lifshitz E. M. The Classical Theory of Fields. 4th edition, translated by Morton Hammermesh, Butterworth-Heinemann, 1980.

Vol. 4, 2011 ISSN 1654-9163

— THE — ABRAHAM ZELMANOV JOURNAL

The journal for General Relativity, gravitation and cosmology

— TIDSKRIFTEN — ABRAHAM ZELMANOV

Den tidskrift för allmänna relativitetsteorin, gravitation och kosmologi

Editor (redaktör): Dmitri Rabounski
Secretary (sekreterare): Indranu Suhendro

The Abraham Zelmanov Journal is a non-commercial, academic journal registered with the *Royal National Library of Sweden.* This journal was typeset using LaTeX typesetting system. Powered by BaKoMa-TeX.

The Abraham Zelmanov Journal är en ickekommersiell, akademisk tidskrift registrerat hos *Kungliga biblioteket.* Denna tidskrift är typsatt med typsättningssystemet LaTeX. Utförd genom BaKoMa-TeX.